

Math Chapter 5 Problems *silvaAndrew*

asilva14

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1 Circles

Find the difference between the points.

2. $(3, 3)$ and $(-3, -2)$

$$d = 6^2 + 5^2 = c^2$$

$$d = 61^2$$

$$d \approx 7.81$$

I found this one easy, I just used Pythagorean Theorem by using the length and height of the distance between the points to find the hypotenuse of the triangle to find the actual distance between the points.

Write an equation of the circle.

4. Center $(-9, 9)$ with a radius of 16

$$(x + 9)^2 + (y - 9)^2 = 16$$

This one was simply plugging in the information given into the equation of a circle. The thing I have to be careful for is the sign of the number when plugin into the equation.

Write an equation of a circle.

6. Center at $(3, -7)$: that passes through the point $(15, 13)$.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(15 - 3)^2 + (13 + 7)^2 = r^2$$

$$144 + 400 = r^2$$

$$544 = r^2$$

$$\text{So, } (x - 3)^2 + (y + 7)^2 = 544$$

I found this one easy, i was plugging it is just plugging in the center point

and the other point given to find the radius. from there you can write the equation of the circle.

Write the equation for a circle with two points on the diameter.

8. Points (-3,3) and (5,7).

$$r^2 = (x - h)^2 + (y - k)^2$$

Use midpoint formula: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
 $(\frac{-3+5}{2}, \frac{3+7}{2}) \rightarrow (\frac{2}{2}, \frac{10}{2}) \rightarrow (1,5)$

$$\begin{aligned} (-3 - 1)^2 + (3 - 5)^2 &= r^2 \\ -16 + (-4) &= r^2 \\ (x - 1)^2 + (y - 5)^2 &= -20 \end{aligned}$$

I found this one easy as well. You need to know the mid point formula to find the center point of the circle. From there you can use the center point and one of the points given to find the radius of the circle. Then from there you plug in the center point and the radius into the equation of a circle.

Sketch a graph of the circle. 10. $(x + 1)^2 + (y - 2)^2 = 16$

I have it completed on mathematica, but because I have a mac it wont save properly to my computer. I've tried uploading it to dropbox to see if I could get it to work that way but still no luck. I have no idea how to get the graph from mathematica over to Latex. I have asked for help in class but the information I was given does not help this solution. I had confusion at first but I asked questions in class that helped me figure out how to graph the circle. The parts that I struggled with this problem was plugging the problem into the mathematica. I have not worked with Mathematica too much so I am not fully comfortable with the language. I did not know about the ContourPlot and I forgot about having to put two = in the problem.

Find the x-intercepts of a circle.

12. Center at (2,3) and a radius of 4.

Same thing as the last one. After asking questions in class I was able to graph the circle and find the x-intercepts. But because I have a Mac I can seem to figure out how to save it from the other firmware to the Mac OS. I've tried drop box but that still didn't work. I'm going to ask questions in class to see if you have any idea.

$$\begin{aligned} 12. (x - (-2))^2 + (y - 0)^2 &= 3^2 \text{ so, } (x + 2)^2 + (y^2) = 9 \\ \text{The line intersects the circle when } y &= 2x + 5 \\ (x + 2)^2 + (2x + 5)^2 &= 9 \quad x^2 + 4x + 4 + 4x^2 + 20x + 25 = 9 \end{aligned}$$

$$5x^2 + 24x + 20 = 0$$

This quadratic formula gives us ≈ -3.7266 and ≈ -1.0734 . Plugging these into the linear equation gives us the two points $(-3.7266, -2.4533)$ and $(-1.0734, 2.8533)$ of which on the second one is in the Second Quadrant.

I had to look the solution up for this one. I was able to figure out the circle equation but from there I was no aware where to go next. After looking at the solution, I realized you plug in the equation for the line for y. I would not have known to use the quadratic formula either.

17. Transmitter origin $(0,0)$ so, $x^2 + y^2 = 53^2$

$$y = \frac{-35}{37}(x - 74) = -35/37x + 70$$

$$y = -0.945946x + 70$$

$$\text{So, } x^2 + (-0.945946x + 70)^2 = 2809$$

$$x^2 + 0.894814^2 - 132.43244x + 4900 = 2809$$

$$189481x^2 - 132.43244x + 2091 = 0$$

After applying the quadratic formula, $x \approx 24.0977$ and $x \approx 45.7944$ The points of intersecting are $(24.0977, 47.2044)$ and $(45.7944, 26.6810)$

$$d = \sqrt{(45.7944 - 24.0977)^2 + (26.6810 - 47.2044)^2} \approx 29.87$$

I had to look up the solution to this one too. I was able to figure out up till the part dealing with the quadratic formula. I understood when to substitute y into the equation but I was not sure if the quadratic formula was needed. Once again it comes back to remembering when to use certain formulas.

2 Angles

6. Convert the angle $\frac{11\pi}{6}$ from radians to degrees.

$$\left(\frac{11\pi}{6}\right)\left(\frac{180}{\pi}\right) = 330^\circ$$

I thought this one was easy. By crossing out like terms it helped make the multiplication in the conversion easier.

12. Find the angle between 0 and 2π in radians that is co terminal with the angle $\frac{17\pi}{3}$.

$$\left(\frac{17\pi}{3} - 2\pi\right) = \left(\frac{17\pi}{3} - \frac{6\pi}{3}\right) = \frac{11\pi}{3}$$

After looking up a few videos online I was able to figure this one out on my own. I was confused on how to start it. But I realized you can turn 2π into a fraction with the same denominator as the co terminal. then from there you can subtract it and solve for the answer.

16. On a circle of a radius of 6 feet, find the length of the arc that subtends a central angle of 1 radian.

$$r = 6ft$$

X or the central angle = 1 radian

$$\text{So, } Xr = (6)(1) = 6 \text{ feet}$$

I had to look up the solution to this one. After looking at the solution it looks simple, but I am still confused on why we multiplied them together to get the answer.

25. How many revolutions per minute do the wheels make?

32in diameter is traveling 60 mph. So,

32in diameter is traveling 63360 inches/min. So,

$63360/16 = 3960$ radians per minute.

$$\frac{3960}{2\pi} = 630.2535 \text{ revs per min}$$

After using a calculator to find out the inches/min translation of the speed, it was much easier to figure out the rest. From there you just divide by the radius to find the radians per minute. Then from there divide by 2π to get the revs per min. Understanding what and how to translate the speed to is what was messing me up the most. But taking my time allows me to use my resources around me to figure out what I'm stuck on. After checking with the solution manual I realized that I divided by the diameter instead of the radius. Small mistakes like that is where I mess up most.

$$31. \quad w = \frac{2\pi}{t} = \frac{2\pi}{24 \text{ hrs}} = \frac{\pi}{12}$$

$$v = rw \text{ so, } (3960 \text{ miles}) \left(\frac{\pi \text{ rad}}{12 \text{ hrs}} \right) = 1036.73 \text{ miles}$$

I found this one much easier than the last. I feel like because there were less transitions that had to be done so there was less to confuse me. It was not as complicated as the last one, and I feel like seeing the answer to the last one helped me when completing this one.

3 Points on Circles Using Sin and Cos

2. Find the quadrant in which the terminal point determined by t lies if. A.) $\sin(t) < 0$ and $\cos(t) > 0$

Quadrant 4

B.) $\sin(t) > 0$ and $\cos(t) > 0$

Quadrant 1

At first I was having a tough time with this one. I understood the problem, however I was reading the signs backwards. Once I slowed down and took my time when reading the question, I understand that it was looking for the quadrants on the graph that the signs of Sin and Cos matched the signs given. For this one remembering that Sin is y and Cos is x is very important, as well as remembering which quadrants x and y are positive and negative in.

3. The point P is on the unit circle. If the y-coordinate of P is $\frac{3}{5}$, and P is in quadrant II, find the x coordinate.

$$\frac{3}{5} = \frac{3}{5}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\frac{9}{25} + \cos^2(\theta) = 1$$

$$\begin{aligned} \cos^2(\theta) &= \frac{25}{25} - \frac{9}{25} = \frac{16}{25} \\ \cos(\theta) &= \pm \frac{4}{5} \end{aligned}$$

We are in quadrant 2, Cos is negative in Quadrant 2. So, $\cos(\theta) = -\frac{4}{5}$

I had to look up the solution to this one, I had no clue where to begin with this one. I tried looking up the unit circle but that was no use. After looking up the solution, I realized you need to use Pythagorean Theorem. It makes sense because Sin represents the y and then the hypotenuse would be 1. Then you can solve for x or the Cos. I also have to be careful and make sure the signs match the quadrant that it is in.

5. If $\cos(\theta) = \frac{1}{7}$ in the 4th Quadrant, find $\sin(\theta)$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin^2(\theta) + \frac{1}{7^2} = 1$$

$$\sin^2(\theta) + \frac{1}{49} = 1$$

$$\frac{49}{49} - \frac{1}{49} = \sin^2(\theta) \text{ or } \pm \frac{\sqrt{48}}{7}$$

$\sin(\theta)$ is negative in the 4th Quadrant so, $-\frac{\sqrt{48}}{7}$

I solved this one on my own. When I checked the the solutions to check my answer, I realized I did not simplify it fully. I still do not fully know when to simplify square roots. The more I work with them the more I feel stronger working with them, however I still feel like there is so much I don't know about square roots.

6. If $\cos(\theta) = \frac{2}{9}$ in the 1st Quadrant, find $\sin(\theta)$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin^2(\theta) + \left(\frac{2}{9}\right)^2 = 1$$

$$\sin^2(\theta) + \left(\frac{4}{81}\right) = 1 \text{ So,}$$

$$\frac{81}{81} - \frac{4}{81} = \frac{77}{81} \text{ So,}$$

$$\sin(\theta) = \pm \frac{\sqrt{77}}{81}$$

\sin in the first Quadrant is Positive so, $= \frac{\sqrt{77}}{81}$

After the last one this one was much simpler. Working through the problem was much easier and as well as starting it. Knowing to use Pythagorean Theorem made me not over think the beginning. The only thing I still and not sure one, is if I need to simplify the square root.

11. For each of the following angles, find the reference angle and which quadrant the angle lies in. Then compute sine and cosine of the angle.

A.) $\frac{5\pi}{4}$ which has a reference angle of $\frac{\pi}{4}$

$\frac{5\pi}{4}$ is in Quadrant 3, Sin is Negative and Cos is Negative.

$$\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

B.) $\frac{7\pi}{6}$ which has a reference angle of $\frac{\pi}{6}$

$\frac{7\pi}{6}$ is in Quadrant 3, Sin is Negative and Cos is Negative.

$$\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

C.) $\frac{5\pi}{3}$ which has a reference angle of $\frac{\pi}{3}$
 $\frac{5\pi}{3}$ is in Quadrant 4, Sin is Negative and Cos is Positive.

$$\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

D.) $\frac{3\pi}{4}$ which has a reference angle of $\frac{\pi}{4}$
 $\frac{3\pi}{4}$ is in Quadrant 2, Sin is Positive and Cos is Negative.

$$\sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

After taking my time with this one, I was able to figure it out. After figuring out what a reference angle was, I then made sure all the radians given were common ones on the Unit circle. The thing I had to be careful with was when I was calculating I had to be careful that the answer was given the correct sign, positive or negative, depending on if it was sin or cos and depending on the Quadrant it was in. Also it is important to remember that sin matches up with y and cos represent x. I then used the unit circle to find the answer.

13. Give exact values for $\sin(\quad)$ and $\cos(\quad)$ for each of these angles.

A.) $-\frac{3\pi}{4}$

$$\sin\left(-\frac{3\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{3\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

B.) $-\frac{23\pi}{6}$

$$\sin\left(-\frac{23\pi}{6}\right) = -\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(-\frac{23\pi}{6}\right) = \cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

C.) $-\frac{\pi}{2}$

$$\sin\left(-\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1$$

$$\cos\left(-\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

C.) 5π

$$\sin(5\pi) = \sin(\pi) = 0$$

$$\cos(5\pi) = \cos(\pi) = -1$$

This one took me a little while to understand. I took my time and was able to complete it. I figured it would be similar to the last one, I was just confused on what it meant by find the exact value. But once I realized you need to find the reference angle, then use the unit circle to find the exact values. The last

two tricked me but I tried to not over think it and I think that I figured it out. Another problem to note that cos represents x and sin represents y.

4 The Other Trigonometric Functions

2. If $\theta = \frac{7\pi}{4}$ find exact values for $\sec(\theta)$, $\csc(\theta)$, $\tan(\theta)$, $\cot(\theta)$.

$$\sec\left(\frac{7\pi}{4}\right) = \frac{1}{\cos\left(\frac{7\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\csc\left(\frac{7\pi}{4}\right) = \frac{1}{\sin\left(\frac{7\pi}{4}\right)} = -\frac{1}{\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

$$\tan\left(\frac{7\pi}{4}\right) = \frac{\sin\left(\frac{7\pi}{4}\right)}{\cos\left(\frac{7\pi}{4}\right)} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{-\sqrt{2}}{\sqrt{2}} = -1$$

$$\cot\left(\frac{7\pi}{4}\right) = \left(\frac{1}{\tan\left(\frac{7\pi}{4}\right)}\right) = \frac{1}{-1} = -1$$

I was a bit confused with this one at first. I knew it was going to be similar to the last one. I also knew that \sec is \cos^{-1} which is also $\frac{1}{\cos(\theta)}$. This helped me to solve for the exact values. For \tan I had to remember that you put \sin over \cos because the hypotenuse is opposite over adjacent. \sin is represented as opposite and \cos represents adjacent. From there you can use \tan to find the \cot exact value.

10. If $\sin(\theta) = \frac{2}{7}$, and θ is in quadrant II, find $\cos(\theta)$, $\sec(\theta)$, $\csc(\theta)$, $\tan(\theta)$, $\cot(\theta)$.

$$\text{Adj.} = \sqrt{\text{hyp}^2 - \text{opp}^2}$$

$$\text{Adj.} = \sqrt{7^2 - 2^2}$$

$$\text{Adj.} = \sqrt{49 - 4}$$

$$\text{Adj.} = \sqrt{45}$$

$$\cos = \frac{\text{adj}}{\text{hyp}} \quad \cos = \frac{\sqrt{45}}{7}$$

$$\tan = \frac{\text{opp}}{\text{adj}} \quad \tan = \frac{2}{\sqrt{45}}$$

$$\cot = \frac{\text{adj}}{\text{opp}} \quad \cot = \frac{\sqrt{45}}{2}$$

$$\csc = \frac{\text{hyp}}{\text{opp}} \quad \csc = \frac{7}{2}$$

$$\sec = \frac{\text{hyp}}{\text{adj}} \quad \sec = \frac{7}{\sqrt{45}}$$

I found this one easy. The first part is what confused me however After that it is just remembering Soh Cah Toa, then plugging in.

Simplify

17. $\csc(t)\tan(t)$

$$\frac{1}{\sin(t)} * \frac{\sin(t)}{\cos(t)} = \frac{1}{\cos(t)} = \sec(t)$$

I had to look up the solution to this one. At first It seemed easy, But I didn't know how to start it. After looking at the solution, I should've not thought about it too much and just applied what I know about \csc and \tan , and replaced them with \sin and \cos . After looking at the solution my confusion is gone. I understand how to solve problems like this.

Prove the Identities.

$$29. \sec(a)\cos(a) = \sin(a)\tan(a)$$

$$ec(a)\cos(a) = \frac{1}{\cos(a)} - \cos(a)$$

$$sec(a)\cos(a) = \frac{1}{\cos(a)} - \frac{\cos^2(a)}{\cos(a)}$$

$$sec(a)\cos(a) = \frac{\sin^2(a)}{\cos(a)}$$

$$sec(a)\cos(a) = \sin(a) * \frac{\sin(a)}{\cos(a)}$$

$$sec(a)\cos(a) = \sin(a)\tan(a)$$

I had to look up the solution to this one. I was not sure what the question was asking by "prove the Identifies". After looking at the solution I realized that it was asking prove the equation was correctly equal by showing it in another form.

5 Right Triangle Trigonometry

2. In the triangle find find $\sin(A), \cos(A), \tan(A), \sec(A), \csc(A), \cot(A)$.

$$\text{hyp} = 4^2 + 10^2 = \sqrt{116}$$

$$\text{adj} = 4$$

$$\text{opp} = 4$$

$$\sin(a) = \frac{10}{\sqrt{116}}$$

$$\cos(a) = \frac{4}{\sqrt{116}}$$

$$\tan(a) = \frac{10}{4} = \frac{5}{2}$$

$$\csc(a) = \frac{\sqrt{116}}{10}$$

$$\sec(a) = \frac{\sqrt{116}}{4}$$

$$\cot(a) = \frac{4}{10} = \frac{2}{5}$$

I found this one easy. After finding the Hyp, it is just simply plugging in the adj, opp, and hyp into each equation depending on what trig function you are using. Remembering Soh Cah Toa really helps with problems like these.

4. Solve for the unknown sides and angles.

$$180^\circ - 90^\circ - 60^\circ = 30^\circ = A$$

$$\sin(60^\circ) = \frac{10}{c} = \frac{10}{\sin(60^\circ)} = \frac{10}{\frac{\sqrt{3}}{2}} = 20\sqrt{3} = \frac{20\sqrt{3}}{3} = c$$

$$\tan(60^\circ) = \frac{10}{a} = \frac{10}{a} = 1 \frac{10}{\tan(60^\circ)} = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3} = a$$

This one took me a bit longer than the last. I had to take my time with this one. The part I struggled with the most was simplifying the answer for the the sides. The more I work with using square roots the more I understand more about them. Using them in different types of problems helps me figure out how to use them in different situations.

10. A 23-ft ladder leans against a building so that the angle between the ground and the ladder is 80° . How high does the ladder reach up the side of the building?

$$\sin(80^\circ) = \frac{a}{23}$$

$$23 * \sin(80^\circ) \approx 22.651 \text{ feet}$$

I was confused with this one at first, I knew how to set it up and I knew to use Sin. The only thing I was messing up was to remember to multiply the $\sin(80^\circ)$ by the height of the ladder to find the height the ladder reaches.

20. Find the length x.

$$\tan(50^\circ) = \frac{85}{a} = \frac{85}{\tan(50^\circ)}$$

$$\tan(36^\circ) = \frac{85}{a} = \frac{85}{\tan(36^\circ)}$$

$$\text{So, } \frac{85}{\tan(50^\circ)} + \frac{85}{\tan(36^\circ)} =$$

$$71.3235 + 116.993 \approx 188.316$$

$$x = 188.316$$

This one was similar to the last one. I understood the process needed to complete it. I just had trouble going to add the two lengths. But After realizing to use a calculator it made it much simpler.

23. How high is the plane above the top of the mountain (when it passes over)? What is the height of the mountain?

$$PT = \left(\frac{100}{1h}\right)\left(\frac{1h}{60min}\right)(5min) = \frac{25}{3}44000 \text{ ft}$$

$$\Delta PTL = \sin(20^\circ) = \frac{TL}{PT} \text{ so, } TL = PT(\sin(20^\circ)) = 44000(\sin(20^\circ))$$

$$\cos(20^\circ) = \frac{PL}{PT} \text{ So, } PL = PT(\cos(20^\circ))$$

$$\Delta PEL, \tan(18^\circ) = \frac{EL}{PL} \text{ So, } EL = PL(\tan(18^\circ)) =$$

$$44000(\cos(20^\circ))(\tan(18^\circ)) \approx 13434.284 \text{ ft}$$

$$TE = TL - EL = 44000(\sin(20^\circ)) - 44000(\cos(20^\circ))(\tan(18^\circ)) =$$

$$44000(\sin(20^\circ) - \cos(20^\circ)(\tan(18^\circ))) \approx 1614.602$$

$$TE = 1614.602 \text{ ft } EL = 13434.284 \text{ ft } \text{Height of the mountain} = 13434.284 + 2000 = 15434.28 \text{ ft}$$

This one was much more complicated than these past few. But after taking my time with it I was able to complete it. We also went over this one in class so that helped me a lot in knowing how to start it. An important thing to remember is that the the Answer for EL is not the total height of the mountain, you have to add the elevation that you started at to it.