MATH Chapter1 Problems

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1   Function and Function Notation

For each of the Functions evaluate f(-2), f(-1), f(0), f(1, and f(2).

27.

$f(x) = 3 + \sqrt{x+3}$

$f(-2) = 3 + \sqrt{-2+3} = 3 + 1 = 4$

$f(-1) = 3 + \sqrt{-1+3} = 3 + \sqrt{2} = 3 + 1.4142 = 4.4142$

$f(0) = 3 + \sqrt{0+3} = 3 + \sqrt{3} = 3 + 1.7321 = 4.7321$

$f(1) = 3 + \sqrt{1+3} = 3 + \sqrt{4} = 3 + 2 = 5$

$f(2) = 3 + \sqrt{2+3} = 3 + \sqrt{5} = 3 + 2.236 = 5.236$

Whenever I see a square root in a problem I tend to stress out because I feel like answers must come out to a whole number but that is not the case. I find my self second guessing any answer that is not a whole number. I found the algebra easy in this problem though. Even with the square root. I tried picking a problem I am unfamiliar with to try and get more familiar with square roots.

32.

$f(x) = \frac{x-2}{x+2}$

$f(-2) = \frac{-2-2}{-2+2} = \frac{-4}{0} = Undefined$

$f(-1) = \frac{-1-2}{-1+2} = \frac{-3}{1} = -3$

$f(0) = \frac{0-2}{0+2} = \frac{-2}{2} = \frac{-1}{1} = -1$

$f(1) = \frac{1-2}{1+2} = \frac{-1}{3}$

$f(2) = \frac{2-2}{2+2} = \frac{0}{4} = 0$

Another thing I tend to avoid in equations are fractions. I always second guess myself on fractions as well. I tend to forget to reduce as well as keeping track of signs while dealing with fractions. Luckily the algebra wasn't difficult in this problem so I could keep track of the signs easily. Another thing I get tripped up on are when answers come out to undefined. I don't always remember that things can be undefined.

35.

Suppose $f(x) = x^2 + 8x - 4$

A.$f(-1) + f(1)$

$f(-1) + f(1) = -1^2 + 8(-1) - 4 + 1^2 + 8(1) - 4$

$f(-1) + f(1) = 1 + (-8) - 4 + 1 + 8 - 4 = -6$

$f(-1) + f(1) = -6$

B. $f(-1) - f(1)$
$f(-1) - f(1) = -1^2 + 8(-1) - 4 - 1^2 + 8(1) - 4$
$f(-1) - f(1) = 1 + 8 - 4 - 1 + 8 - 4 = -8$
$f(-1) - f(1) = -8$

What I found difficult in this problem was I wasn't sure what it was asking at first. But I soon realized that you had to find the F(-1) and the f(1) separately then add then together. However I realized you can just do it all in one equation. Another thing I had to be careful with was making sure the signs carried correctly. The algebra was easy for me, besides having to take my time with the signs.

44.

Write the equation of the circle centered at (9, -8) with a radius of 11.
$(x - 9)^2 + (y - (-8))^2 = 11$

I had trouble with this problem because I thought we were solving for something. Once I realized we were looking for the equation and not solving for something I figured it out. How ever I do need to remember that the x is the opposite of what it gives you. I also need to slow down when reading problems, I tend to try and get things done as quick as possibly because that's what it felt like teachers wanted in high school, but that causes me to overlook important information.

2 Domain and Range

16.

Find the domain of the Function.
$f(x) = \frac{\sqrt{x+5}}{x-6}$
$\{x | x E, x \neq 6\}$

I can't figure out how to write the answer properly. But I choose this one because it has both square roots and fractions in it. I want to get more experience in working with them in different equations to figure out new ways to learn how to properly calculate and use them in problems.

22.

Given each Function evaluate: f(-1), f(0), f(2), and f(4).
$f(x) = \begin{cases} 4 - x^3, & if x < 1 \\ \sqrt{x + 1}), & if x \geq 1 \end{cases}$
$f(-1) = 4 - (-1)^3 = 4 - (-1) = 5$
$f(0) = 4 - 0^3 = 4$
$f(2) = \sqrt{2 + 1} = \sqrt{3}$
$f(4) = \sqrt{5 + 1} = \sqrt{5}$

Once again the square root in the answers confused me, but I figured I could just leave it like that since it looks cleaner then converting it to a decimal. I also got confused what it meant by the "if", but i realized it meant and figured it out soon after. The algebra was easy though.   33.

Sketch a graph of each piece wise function.

$$f(x) = \begin{cases} x^2, & if x < 0 \\ x+2, & if x \le 0 \end{cases}$$

I have no clue on how to graph on Mathematica. I've sat here for about an hour looking at videos, the website, and other online websites and i still have no clue why it is not working for me. However other then trying to create the graph online, I was also confused on how we were supposed to graph the equation without any points. So I had to look at the solution for this one. But once I saw the solution I realized what I was confused about. I have a problem where if the equation does not look familiar then I get nervous and over think what the problem is really asking. The thing I found easy on this problem was deciding if the circle needed to be open or closed depending on the type the ¡,¿, or equal to signs were.

36.

$$f(x) = \begin{cases} -3, & if x \le 2 \\ x-1, & if -2 < x \ge 2 \\ 0, & if x > 2 \end{cases}$$

Once again I can not figure out creating a graph in Mathematica. After doing the last one I understood the problem, so I knew where to start. However I did find it a little more difficult because it had 3 different lines rather then 2. So I don't know If my connections are correct but I think I have it correct.

3 Rates of Change and Behavior of Graphs

9.Find the average rate of change of the function on the given interval. $k(t) = 6t^2 + \frac{4}{t^3}$ on [-1,3]

$k(-1) = 6(-1)^2 + \frac{4}{-1^3} = 6 + \frac{4}{-1} = 6 + (-4) = 2$

$k(3) = 6(3)^2 + \frac{4}{3^3} = 6(9) + \frac{4}{27} = 54 + \frac{4}{27} = \frac{1,458}{27} + \frac{4}{27} = \frac{1,462}{27}$

$$\frac{k(3) - k(-1)}{3 - (-1)} = \frac{2 - \frac{1,462}{27)}}{4} = \frac{\frac{-1408}{27}}{4}$$

I though the fraction would create more of a confusion for me than it did. However I figured it out. The large negative number in the numerator scares me that it's wrong but I need to stop second guessing myself. Another thing I had to pay attention too in this problem was the negative signs, I had to make sure they carried with the respected number.

10.

$p(t) = \frac{t^2 - 4t + 1}{t^2 + 3}$ on [-3,1]

$p(-3) = \frac{-3^2 - 4(-3) + 1}{-3^2) + 3} = \frac{9 - 12 + 1}{9 + 3} = \frac{-3 + 1}{12} = \frac{-2}{12} = \frac{-1}{6}$

$p(1) = \frac{1^2 - 4(1) + 1}{1^2 + 3} = \frac{1 - 4 + 1}{1 + 3} = \frac{-2}{4} = \frac{-1}{2}$

$\frac{p(1) - p(-3)}{1 - (-3)} = \frac{\frac{-1}{2} - \frac{-1}{6}}{4} = \frac{\frac{-4}{12}}{4} = \frac{-1}{3}$

The functions are getting easier to understand and complete. I was nervous at the end because I got a fraction but it looks like the math is correct.I'm also nervous that I didn't carry a sign when I should have. So I hope all the negative

signs were where they should be. But I tried to slow down and take my time with this one so I think the signs are good. Also, there was a lot of fractions in this one, I though I was going to make a mistake with them as well so I went slow with this one.

15.

$a(t) = \frac{1}{t+4}$ on [9, 9+h]

$a(9) = \frac{1}{9+4} = \frac{1}{13}$

$a(9) = \frac{1}{9+h+4} = \frac{1}{13+h}$

$\frac{\frac{1}{13+h} - \frac{1}{13}}{9+h-9} = \frac{\frac{1}{13+h} - \frac{1}{13}}{h} * \frac{1}{h} = \frac{1}{h(h+13)} - \frac{1}{13h} = \frac{1}{h^2+13h} - \frac{1}{13h} * \frac{\frac{h}{13}+1}{\frac{h}{13}+1} = \frac{1}{h^2+13h} -$

$\frac{\frac{h}{13}+1}{h^2+13h} = \frac{1-\frac{h}{13}-1}{h^2+13h} = \frac{\frac{h}{13}}{h^2+13h} = \frac{h}{13} * \frac{1}{h^2+13h} = \frac{h}{13(h^2+13h} = \frac{h}{13h(h+13} = \frac{h}{13(h+13)}$

I had tolook up the solution for this one. I got to $\frac{\frac{1}{13+h} - \frac{1}{13}}{9+h-9}$ and then didn't know where to go from there. I forgot you're supposed to multiple by $\frac{1}{h}$. I was getting thrown off because my answer kept coming out to $\frac{h}{h}$ and I knew that wasn't correct. Next time I can take my time and try and look at all my options before just jumping to the solution when I'm stuck.

39. $g(t) = t\sqrt{t+3}$

Once again I don't know how to graph online. But I had to look up the solution to this one. Once again I was confused on how to plot without points. I had no clue how to graph it, and even after looking at the solution I am still a bit confused. I need to work on graphing equations and functions.

4   Composition of Functions

3. Find f(g(0)) and g(f(0))

$f(x) = \sqrt{x+4}$

$g(x) = 12 - x^3$

$f(0) = \sqrt{0+4} = \sqrt{4} = 2$

$g(0) = 12 - 0^3 = 12$

$f(g(0)) = \sqrt{12+4} = 4$

$g(f(0)) = 12 - 2^3 = 12 - 8 = 4$

I chose this problem because it has square roots so I wanted to work on square roots. I'm starting to understand and not be scared of squareroots when I see them. The wording of f(g(0))and g(f(0)) was confusingme at first as well. However the more I worked with it that more comfortableI was getting with it. At first I was confused why there was a 0, but then I realized that what you plug in originally.

22. Find f(g(x)) and g(f(x)).

$f(x) = \frac{1}{x-4}$,

$g(x) = \frac{2}{x} + 4$

$f(g(x)) = \frac{1}{\frac{2}{x}+4-4} = \frac{1}{\frac{2}{x}} = \frac{x}{2}$

$g(f(x)) = \frac{2}{\frac{1}{x}-4} + 4 = \frac{6}{\frac{1}{x}} * \frac{\frac{x}{1}}{\frac{1}{1}} = 6x$

I thought I had this one easily, however I realized I had made a mistake. I got stuck on how to get the x out of the denominator. I know it's a simple confusion that I should get over but I have struggled with understanding fractions all throughout schooling. To work on them I should take my time,while doing any problem having to do with fractions.

25. $f(x) = |x|$, $g(x) = 5x + 1$

$f(g(x)) = |5x + 1|$

$g(f(x)) = 5|x| + 1$

I chose this one because haven't worked with absolute values yet normuch at all. I know a little about them but not much. I ended up having to double check my answer with the solutions because it seemed too simple. I need to start choosing more problems that involve them because I need to become more firmiliar with them.

28. $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$, $h(x) = x + 3$

$f(g(h(x))) = (\frac{1}{x+3})^2 + 1 = \frac{1^2}{x^1 + 3^2} + 1 = 1 + \frac{1}{x^2 +}$

After I got the answer I second guessed myself and tried to continue to solve the problem. I think what my problem with second guessing myself is that when I look at an answer like that, my brain tries and thinks of any rules I've learned even if it has nothing to do with this problem. Then I try to apply those rules to get try and reduce it to a smaller or single number. I need to get in the habit of understnading that different problems have different looking solutions.

5 Transformation of Functions

12. f(x)=—x— shifted down 3 units and right 1 unit

$f(x) = (|x| - 1) - 3$

I found this one easy because I still remember parts of these from high school.I remembered vertical movement is always outside and horizontal movement is inside the parenthesis. I also luckily remembered that when you move something horizontal, in the equation it is the opposite, so if its moving right you make it negative or subtract it from x.

33.$f(x) = 6^x$

A. Reflecting f(x) over the x and y axis.

$f(x) = -6^{-x}$

B. Reflecting f(x) about the x-axis shifting left 2 units and down 3 units.

$f(x) = -6^{x+2} - 3$

When I originally did part A I forgot to make the 6 negative, but when I did part B, I realized that I needed to make the 6 negative in part A as well. Once again going too quickly and miss important parts of the problem. I need to slowdown when doing math because I'm not welled skilled in it. But that's why i try and get it done quickly because I'm not skilled in it.

40.

A.

$f(x) = (x - 2)^2 = x^2 - 2^2 = \sqrt{x^2} - \sqrt{4} = x - 2$

$-1(x - 2) = -1(-x + 2) = Odd$

B.
$g(x) = 2x^4 = 2(-x)^4 = 2(x) = 2x = Even$
C.
$h(x) = 2x - x^3 = 2(-x) - (-x)^3 = -2x - (-x) = -x = Neither$

I'm not sure if I got these correct. I struggled on whether or not I was supposed to try and get the x alone. I was honestly struggling with the algebra on this one I believe. I'm also not sure how to tell if it's even, odd,or neither, I kinda guessed a little. I need to ask more questions about ones like these.

67.
$f(x) = 4(x + 1)^2 - 5 =$ the function is decreasing on the Interval $x > -1$

I can't figure how to graph it on Mathematica. I had no clue how to graph this in general though. Even looking up the answer didn't really help me understand it. I need to focus in graphing because I lacking a lot of skills in graphing.

6 Inverse Functions

14. Find $f^{-1}(x)$ of the function $x - 5$
$f(x) = x + 5 = f^{-1}(x) = x - 5$

I had to check the solution for this one, I thought I knew how to do it, but once I looked at the solution I realized I had no clue. I need too pay attention and read more carefully next time I am going over these. I also need to watch more videos to get a better understanding of problems like these.

16.
Find $f^{-1}(x)$ of the function $3 - x$
$f(x) = 3 - x = f^{-1}(x) = 3 + x$

I had to check the solutions for this one as well, I just don't seem to fully understand. When I got to look up how to do these on Youtube and google I just get stressed out because I don't know where to begin with their steps. Usually Youtube videos help a lot but this time I just don't seem to be understanding.

19.
f(x)=(x+7)^2
Domain so f(x) is non decreasing and one to one would be $x < -7$
Inverse of the Function is $f^{-1}(x) = \sqrt{x - 7}$

I found this one to be easy, Once I graphed it I knew the domain. However at first I read decreasing instead of non-decreasing, so I had $x > -7$, however I realized my mistake because I started to go through the problem after I finished it and I noticed it. I need to slow down when reading questions, I get too a head of my self.

20.
$f(x) = (x - 6)^2$
$x > 6$
The inverse function of this function would be $f^{-1}(x) = \sqrt{x + 6}$

I found this one easy because it was very similar to the last one. I chose to do a similar one to try reinforce my experience with these. I graphed it well but

I don't know if I'm correct. I'm just trying not to second guess my self on every problem I come across. I need to work on my graphing.